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MATHEMATICAL PRELUDE

Another Proof of the Two-sidedness of Matrix Inverses

Oskar Maria Baksalary[1] and Götz Trenkler[2]

Let A and B be $n \times n$ complex matrices and let I_n denote the identity matrix of order n. Inspired by [1] and [2], we provide a concise proof of the known fact that when $AB = I_n$, then $BA = I_n$.

Let $AB = I_n$ and let rk(.) be the rank of a matrix argument. Then $n = \mathrm{rk}(I_n) = \mathrm{rk}(AB) \leqslant \mathrm{rk}(A) \leqslant n$. Hence $\mathrm{rk}(A) = n$. But $\mathrm{rk}(A) = \mathrm{rk}(A^\dagger A)$, where $A^\dagger$ denotes the Moore–Penrose inverse of A. Since $A^\dagger A$ is idempotent and an idempotent matrix of full rank necessarily coincides with the identity matrix, we conclude that $A^\dagger A = I_n$. On the other hand, the equation $AB = I_n$ has the solution for B as $B = A^\dagger + (I_n - A^\dagger A)Z = A^\dagger$ for some $n \times n$ complex matrix Z. In consequence, $BA = A^\dagger A = I_n$.

References

[1] R.A. Beauregard, A short proof of the two-sidedness of matrix inverses, *Mathematics Magazine* 80 (2007) 135–136.

[2] F. Sandomierski, An elementary proof of the two-sidedness of matrix inverses, *Mathematics Magazine* 85 (2012) 289.

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